**PREDICTION OF WELL LOG INTERPRETATION PARAMETERS   
 AN INDEPENDENT STUDY**

E. R. Crain, P. Eng.,   
J.C. Sproule and Associates Ltd., Calgary, Alberta (when written)  
Digital Technology Pty. Ltd., Sydney, Australia (when published)

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\*\*I had taken a job in Sydney and could not get back to the meeting in Calgary.*

**ABSTRACT**Methods for the prediction of the response of density, acoustic velocity and neutron logs to mixtures of gases, liquids and solids, have been compiled in this paper. The methods are based on knowledge of the elastic constants, chemical makeup, and other physical properties of the material; these data are available from various handbooks or other sources.

Examples are given to illustrate the methods and to demonstrate, with familiar materials, the accuracy of the predictions.

**INTRODUCTION**  
Log analysts recognize that all the various rocks, minerals and fluids surrounding the logging device affect its response. With the advent of modern computer technology as applied to log interpretation, we are now able to take into account many of these variables. It is necessary, therefore, to have an adequate description of these variables, in a form which can be readily understood by both the log analyst and the commonly available log interpretation programs.

This paper is an attempt to lay out, in a systematic manner, techniques which the log analyst can use to calculate suitable log interpretation parameters from the physical or chemical properties of the rocks and fluids which he feels are likely to be present in the formations traversed by the well logs. The parameters are then used in multiple porosity interpretation methods (i.e. simultaneous solution methods), linear programming techniques (i.e. trial and error methods) or intuitively by the log analyst as he performs a normal deterministic log analysis.

Multiple porosity techniques for quantitative determination of porosity, fluid content and lithology, as well as exploration for hydrocarbons, evaporites, and other non-metallic materials, are now well established. Several papers covering the field are listed in the references (1, 2, 3, 4, 5, and 6). Techniques using linear programming methods to reduce large quantities of data and constraints have been used with success (7). The use of one or two porosity logs for liquid saturation calculations and gas storage reservoirs have been explained in two papers (8, 9) so the subject is well documented and will not be explained further here

All the parameters needed can indeed be calculated from the physical properties of the materials. Much data is available in handbooks (14) for this purpose and a large volume of data from many sources was recently compiled privately (15), which could be made available to interested parties on request.

The derivation of the interpretation parameters from these data will be discussed below under the headings of density, velocity and neutron response. Examples are shown, using familiar materials, to illustrate each technique.

**DENSITY OF GASES**An important concept is the non-ideal gas law. Simply stated it says:  
    1: (P1 \* V1) / (Z1 \* T1) = (P2 \* V2) / (Z2 \* T2)

Where:  
  P = pressure (psia or Kpa)  
  T = absolute temperature (degrees R or K)  
  V = volume (cubic feet or cubic meters)  
  Z = gas supercompressibility factor (unitless)

Subscripts 1 and 2 represent data at states 1 and 2, respectively. The concept of a supercompressibility factor, which varies with temperature and pressure, is used considerably in the derivation of coefficients for various natural gas mixtures.

The composite supercompressibility factor of a mixture is obtained from:   
    2: Zmix = Sum (Zi \* Yi)

Where:  
  Yi = mole fraction of the ith component  
  Zi = supercompressibility factor of the ith component  
  Zmix = supercompressibility factor of the mixture

Z factors for components of gas can be found in various handbooks or from most laboratory fluid analysis reports.

Densities of gas mixtures can be calculated from individual densities, in the same fashion as Z factors.  
    3: DENSg = Sum (DENSi \* Yi)

Where:  
  DENSg = density of gas mixture (lb/cuft)  
  DENSi = density of ith component (lb/cuft)  
  Yi = mole fraction of the ith component

This applies only to mixtures whose components do not interact. Thus, it is not the method to use for oil with gas in solution, or chemical compounds, or in liquids with dissolved ions. These will be covered later in this Chapter.

Combining equations for any gas, density is then given by:  
    4: DENSg = (P \* M) / (Z \* R \* T)

Where:  
  DENSg = density (lb/cuft or kg/m3)  
  M = average molecular weight of gas (unitless)  
  P = pressure (psia or KPa)  
  R = gas constant  
         = 10.72 (ft^3/ lb-psi / degree R) -English units  
OR R = 0.008314 (m^3 / Kg-KPa / degree K) -Metric units  
  T = absolute temperature (degrees R or K)  
  Z = gas supercompressibility factor (unitless)

Molecular weights of various hydrocarbon gases can be found by summing individual element weights.

To convert a gas density given at surface conditions to downhole conditions, the following equation is used:  
    5: DENSdh = 35.37 \* Pdh \* DENSsurf / (Tdh \* Zdh)

Where:  
  DENS = density (lb/cuft or kg/m3)  
  P = pressure (psia or KPa)  
  T = absolute temperature (degrees R or K)  
  Z = supercompressibility factor (unitless)

Subscripts dh and surf represent downhole and surface conditions, respectively. Standard conditions assumed in these equations are:   
  Psurf = 14.65 psia  or 101 KPa  
  Tsurf = 520’R          or 288’K   
  Zsurf = 1.00

The density of produced gases can be found in laboratory fluid analysis reports. Reports for the formation being interpreted, and surrounding area can be used for both composition and density of gases. The derivation is shown here in some detail, so that the density of any assumed gas could be computed, if needed. Laboratory reports occasionally neglect to state density or Z factors, but not composition.

Example:  
  Given gas composition:  
    C1H2 = 51.8% by weight  
    C2H4 = 28.0% by weight  
   C3H6 = 20.2% by weight

Find mole fraction of each component:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| (1) | (2) | (3) | (4) | (5) |
| Component | Weight % | Molec. | Moles/100 lb | Mole % |
|  | given | Weight | (2 / 3) | (4 / Sum4) |
| C1H2 | 51.8 | 16.04 | 3.23 | 70.0 |
| C2H4 | 28.0 | 30.07 | 0.92 | 20.0 |
| C3H6 | 20.2 | 44.09 | 0.46 | 10.0 |
| Total | 100.0 |  | 4.62 | 100.0 |

NOTE: Gas analyses are usually given as mole %. Therefore, this step is often unnecessary.

**NUMERICAL EXAMPLE**  
Find average molecular weight of this mixture:  
    M = Sum (MWi \* Yi)  
        = 16.04 \* 0.70 + 30.07 \* 0.20 + 44.09 \* 0.10  
    M = 21.65 lbs per lb mole

Calculate specific volume, density, supercompressibility factor, and volume occupied at 1,000 psia, 104 degree F for 1,000 cu. ft. at standard temperature and pressure, of the gas composition given above, for:  
  (1) a perfect gas  
  (2) non-ideal gas

1. Treated as a perfect gas having an average molecular weight of 21.65.  
  a) Supercompressibility factor of the mixture.  
      Z = 1.000 (definition of perfect gas)

  b) Specific volume.  
      V = R \* T / (M \* P) = 1.00 \* (10.72 \* 564) / (21.65 \* 1,000) = 0.2795 cuft/lb

  c) Density  
      1 / V = (21.65 \* 1,000) / (10.72 \* 564) = 3.568 lb/cuft

2. Treated as a real gas using additive volumes and supercompressibility  
factors of individual components:  
  a) Compressibility factor of the mixture (Z).  
      Z = Sum (Zi \* Yi) = 0.918 \* 0.7 + 0.274 \* 0.2 + 0.234 \* 0.1 = 0.721

  b) Specific volume.  
      V = 0.721 \* (10.72 \* 564) / (21.65 \* 1,000) = 0.2015 cuft/lb

  c) Density  
      1 / V = (21.65 \* 1,000) / (0.721 \* 10.72 \* 564) = 4.963 lb/cuft

  d) Volume of 1,000 Scf. at 1,000 psia and 104'F.  
      VOL = 14.65 \* 564 \* 0.721 \* 1,000 / (1,000 \* 520 \* 1.00) = 11.46 cuft

**DENSITY OF LIQUIDS**Oil densities are almost invariably referred to surface conditions. If stock tank oil gravity in API units can be assumed from nearby wells, this must first be converted to density, using the formula given earlier.

The down hole density is calculated from:   
    1: DENSdh = ((GOR \* DENSgas \* 0.178) + DENSoil) / Bo

Where:  
  Bo = formation volume factor of oil and dissolved solution gas (frac)  
  DENSdh = oil density downhole (lb/cuft)  
  DENSgas = solution gas density at surface (lb/cuft)  
  DENSoil = stock tank oil density at surface (lb/cuft)  
  GOR = solution gas to oil ratio (cuft/bbl)

The various factors in the equation must be assumed or taken from fluid analysis from nearby wells. Some average values can be found in handbooks if detailed analysis are unavailable.

Water density can be found from surface measurements and laboratory measurements of the water formation volume factor, or from charts of average data.  
    2: DENSdh = DENSsurf / Bw

Where:  
  Bw = water formation volume factor (fractional)  
  DENSdh = water density downhole (lb/cuft)  
  DENSsurf = water density at surface (lb/cuft)

**NUMERICAL EXAMPLE:**  
  Find the density of oil with the following specifications:  
    temperature = 160'F  
    pressure = 2,000 psia  
    oil gravity = 38 API  
    GOR = 3,000 cuft/bbl  
    DENSgas = 0.002 lb/cuft  
    Bo = 0.728

    DENSoil = 8829.6 / (131.5 + 38.0) = 52.09 lb/cuft (=0.834 gm/cc)  
    DENSdh = (300 \* 0.002 \* 0.178 + 52.09) / 0.728 = 71.70 lb/cuft (= 1.148 gm/cc)

Find the density of water under the same reservoir conditions, having a Bw = 0.90 and a salinity of 200,000 ppm. Assume water with this salinity has a density of 1.139 \* 62.4 = 71.0 lb/cuft.

    DENSdh = 71.0 / 0.90 = 78.8 lb/cuft (= 1.246 gm/cc)

**DENSITY OF SOLIDS**Density of compounds can be calculated in two ways, as shown below. When the  
molar volume is known:  
    1: DENS = M / V

Where:  
  DENS = density (gm/cc or lb/cuft)  
  M = moles (gm or lb)  
  V = molar volume (cc or cuft)

When the molar volume is unknown, but x-ray crystallographic data are available:  
    2: DENS = M \* Z / (Vc \* (6.02 \* 10^23))

Where:  
  DENS = density (gm/cc or lb/cuft)  
  M = moles (gm or lb)  
  Vc = unit cell volume (cc or cuft)  
  Z = number of formula weights per unit cell (unitless)

Unit cell data can be found in the Handbook of Physical Constants, Geological Society of America, New York, 1966.

Well logs do not measure true density but respond to the electron density. The electron density of elements seen by logs is calculated from:  
    3: DENSlog = 2 \* (Z / A) \* DENStrue

Where:  
  A = atomic weight (unitless)  
  DENSlog = log density (gm/cc or lb/cuft)  
  DENStrue = actual density (gm/cc or lb/cuft)  
  Z = atomic number (unitless)

For components or mixtures, the composite Z / A can be calculated from:  
    4: Z / A = Sum (Zi) / M

Where:  
  M = molecular weight of the compound  
  Zi = atomic number of the ith element

Note that Z is used to designate two different variables in the foregoing discussions (atomic number and compressibility factor). Likewise A can stand for atomic number or a constant in the Archie water saturation equation.

Depending on the need, the previous density log equation may require inverting, or the equation may be used as is, to find what the log should read for a given or assumed rock mixture.

**NUMERICAL EXAMPLE**  
  Calculate density log response to pure salt (NaCl):  
  DENS = (23 + 35.4) \* 4 \* 62.4 / (179.42 \* 10^-24 \* 6.02\*10^23) = 135 lb/cuft (= 2.16 gm/cc)

This is based on the molecular weights and the unit cell volume from the Handbook of Physical Constants.

    DENSlog = 2 \* (11 + 17) / 58.4 \* 135 = 129 lb/cu.ft. (= 2.07 gm/cc)

**DENSITY OF MIXTURES**  
The composite density of a mixture of gases, liquids, and solids is obtained by summing the volume-weighted densities of each component:  
    1: DENS = Sum (DENSi \* Vi)

Where:  
  DENS = density of mixture (gm/cc or kg/m3 or lb/cuft)  
  DENSi = density of ith component (gm/cc or kg/m3 or lb/cuft)  
  Vi = volume of ith component (cc or m3 or cuft)

In log analysis terms, the components of a rock are usually rock matrix, shale, oil or gas, and water. The equation becomes:  
    2: DENS = PHIe \* (1 - Sw) \* DENSH  
                   + PHIe \* (Sw) \* DENSW  
                   + Vsh \* DENSSH  
                   + (1 - PHIe - Vsh) \* DENSMA

Where:  
  DENS = density of composite material (gm/cc or kg/m3)  
  DENSH = density of hydrocarbons (gm/cc or kg/m3)  
  DENSMA = density of matrix rock (gm/cc or kg/m3)  
  DENSSH = density of shale (gm/cc or kg/m3)  
  DENSW = density of water (gm/cc or kg/m3)  
  PHIe = porosity (fractional)  
  Sw = water saturation (fractional)  
  Vsh = shale volume (fractional)

This equation is called the Log Response Equation for the density log and forms the basis of density log analysis theory. The equation is rigorous and a law of physics.

Chart, line chart

Description automatically generatedIf a gas analysis or gas density is not available, DENSh for a typical gas in a normal pressure, normal temperature region, such as western Canada, can be chosen from the graph shown at the right.

**NUMERICAL EXAMPLE**  
  Assume data as follows:  
    Vsh = 0.10  
    PHIe = 0.20  
    Sw = 0.25  
    DENSSH = 2.60 gm/cc  
    DENSMA = 2.70 gm/cc  
    DENSW = 1.00 gm/cc  
    DENSH = 0.35 gm/cc  
  
    DENS = 0.20 \* (1 - 0.25) \* 0.35  
= 129 lb/cu.ft. (= 2.07 gm/cc)

**SOUND VELOCITY IN GASES**The only sound wave passed through a gas is the compressional wave. Its velocity is:  
      1: Vgas = 68.4 \* (Ks \* P / DENS)^0.5

Where:  
  DENS = density (lb/cuft)  
  Ks = ratio of specific heats (unitless)  
  P = pressure (psi)  
  Vgas = velocity of sound (ft/sec)

Note that V is a Velocity here, while V is used as Volume elsewhere in this paper.

**Numerical Example:**Find the velocity of sound in a gas with same composition and under the same reservoir conditions as before, given the ratio of specific heats equal to 1.37.  
  Vgas = 68.4 \* (1.37 \* 1000 / 4.936) ^ 0.5 = 1140 ft / sec  
  DTCgas = 10 ^ 6 / 1,140 = 877 msec/ft

**SOUND VELOCITY IN LIQUIDS**  
Again, only the compressional wave is transmitted. Velocity of sound in water or oil can be derived from:   
      1: Vliq = ((144 \* Gc) / (Cw \* DENSliq)) ^ 0.5

Where:  
  Cw = compressibility of liquid (psi^-1)  
  DENSliq = density of liquid at downhole conditions  (lb/cuft)  
  Gc = acceleration of gravity = 32.7 ft/sec^2  
  Vliq = velocity of sound in liquid (ft/sec)

Cw is usually found from laboratory measurements.

**Numerical Example:**Find sound velocity in oil and water from the previous example.  
  Voil = ((144 \* 32.17) / (3.0\*10^-6 \* 71.70)) ^ 0.5 = 4,640 ft/sec  
  DTCoil = (10 ^ 6) / Voil = 206 usec/ft

  Vwater = ((144 \* 32.17) / (2.0\*10^-6 \* 78.80 )) ^ 0.5 = 5,450 ft/sec  
  DTCw = (10^6) / Vwater = 184 usec/ft

**SOUND VELOCITY IN SOLIDS**  
The velocity of longitudinal (compressional) waves in solids can be predicted from the following two equations.  
         1: Vp = 68.4 \* (((K + 4/3 \* N) / DENS) ^ 0.5)  
OR: 1a: Vp = 68.4 \* (((Y \* (1 - N) / (DENS \* (1 - 2 \* N) \* (1 - N)) ^ 0.5)

Where:  
  K = bulk modulus of elasticity (psi)  
  DENS = density (lb/cuft)  
  N = shear modulus or modulus or rigidity (psi)  
  Vp = compressional velocity (ft/sec)  
  Y = Young's modulus (psi)

The transverse (shear) wave velocity is defined by the following two equations:  
        2: Vs = 68.4 \* ((N / DENS) ^ 1/2)  
OR 2a: Vs = 68.4 \* (((Y / DENS) / 2 \* (1 + PR)) ^ 1/2)

Where:  
  DENS = density (lb/cuft)  
  N = shear modulus or modulus or rigidity (psi)  
  PR = Poisson's ratio (unitless)  
  Vs = shear wave velocity (ft/sec)

To translate these formulae into metric, convert density into gm/cc, velocity to Km/sec and the various moduli to megabars, and change the constant terms to 1.0. To convert moduli in megabars to psi, multiply by 6.89 \* 10^-6. To convert megabars to Kilopascals, multiply by 10^4. Also note that 1 Gigapascal (GPa = 10^10 dynes/cm^2).

**VOIGHT and REUSS METHODS**  
Considerable data are available on the elastic constants versus pressure. Three methods are available for compressibility Ce (which is the inverse of the bulk modulus K) versus pressure using equations shown below (Ref 14). The simplest method is:  
  
1: Ce = 1 / K = 6.89 \* 10^-8 \* a + 47.5 \* 10^-16 \* b \* P

Where:  
 Ce = compressibility (psi-1)   
 K = bulk modulus (psi)  
 P = pressure (psi)  
 a = constant (psi-1)  
 b = constant (psi-2)

The constants a and b for particular solids can be found in reference 14.

The Voigt and Reuss schemes for obtaining the elastics constants of quasi-isotropic aggregates of a single material lead to the following relationships, where Cij and Sij refer to the individual crystal, and K and N are the bulk modulus and shear modulus, respectively, of the aggregate:

**Voigt Method**

1: K = (A + 2 \* B) / 3

2: N = (A – B + 3 \* C) / 5

With:  
 3: 3A = (C11 + C22 + C33) \* 14.5 \* 10^6

4: 3B = (C23 + C31 + C12) \* 14.5 \* 10^6

5: 3C = (C44 + C55 + C66) \* 14.5 \* 10^6

**Reuss Method**

1: K = 1 / (3 \* a + 6 \* b)

2: N = 5 / (4 \* a – 4 \* b + 3 \* c)

With:

3: 3a = (S11 + S22 + S33) \* 6.89 \* 10^-8

4: 3b = (S23 + S31 + S12) \* 6.89 \* 10^-8

5: 3c = (S44 + S55 + S66) \* 6.89 \* 10^-8

Where:

a,b,c = intermediate terms (psi^-1)

K = bulk modulus (psi)

Cij = compressibility constants for Voigt method (psi^-1)

N = shear modulus (psi)

Sij = shear constants for the Reuss method (psi^-1)

The Cij and Sij values are obtained from the tables in The Handbook of Physical Constants (reference 14). The modulus of rigidity, is also called the shear modulus (N).

Other coefficients for the aggregate may be obtained from K and N by use of the relationships between the various elastic constants.  **HYDROGEN CONTENT OF GASES**Neutron logs respond primarily to hydrogen in the rocks and fluids near the logging tool. Hydrogen is present in water, oil, gas, and in the clay bound water in shale, or in water of hydration in some minerals, such as gypsum. Some other elements have a minor response, for example chlorine, but modern tool design has attempted to minimize these effects. This discussion deals first with theoretical hydrogen content of materials, then the empirical transforms needed to convert neutron response of real logging tools to apparent porosity.

Since one pound mole of gas always contains 2.74 \* 10^26 molecules, the number of hydrogen atoms in a cubic foot of gas is:  
      1: Hgas = 1.86 \* 10 ^ 25 \* H \* P / (Z \* T)

Where:  
  H = average number of hydrogen atoms per molecule  
  Hgas = number of hydrogen atoms per cubic foot of gas  
  P = pressure (psi)  
  T = temperature (degrees R)  
  Z = gas compressibility factor (unitless)

**NUMERICAL EXAMPLE**  
    Given gas composition:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **(1)** | **(2)** | **(3)** | **(4)** | **(5)** |
| **Component** | **Weight %** | **Molec.** | **Moles/100 lb** | **Mole %** |
|  | **given** | **Weight** | **(2 / 3)** | **(4 / Sum4)** |
| **C1H2** | **51.8** | **16.04** | **3.23** | **70.0** |
| **C2H4** | **28.0** | **30.07** | **0.92** | **20.0** |
| **C3H6** | **20.2** | **44.09** | **0.46** | **10.0** |
| **Total** | **100.0** |  | **4.62** | **100.0** |

The average molecular weight of this mixture is:  
    M = Sum (MWi \* Yi)  
        = 16.04 \* 0.70 + 30.07 \* 0.20 + 44.09 \* 0.10  
    M = 21.65 lbs per lb mole

The number of hydrogen atoms per cubic foot of gas from the above example, held at 2,000 psia and 160 degrees F, can thus be calculated. From the composition, there are an average of 4.8 atoms of hydrogen per molecule.  
    Hgas = 1.86\*10^25 \* 4.8 \* 2,000 / (0.721 \* 620)  
             = 3.99\*10^26 atoms/cuft

**HYDROGEN CONTENT OF LIQUIDS**The number of hydrogen atoms in a hydrogen bearing liquid can be found from the fact that the weight of one hydrogen atom is 1.67\*10^-24 grams. The number of hydrogen atoms can be calculated using the relationship:  
      1: Hwater = 454 \* (1.67\*10^24) \* (H / M) \* DENSw  
                       = 2.72 \* 10 ^ 26 \* (H / M) \* DENSw  
Where:  
  DENSw = density of water at downhole conditions (lb/cuft)  
  Hwater = hydrogen concentration (atoms/cuft)  
  H = average number of hydrogen atoms per molecule  
  M = average molecular weight

The number of hydrogen atoms in oil can be found from a similar equation:  
      2: Hoil = 2.72 \* 10 ^ 26 \* (H / M) \* DENSoil

**NUMERICAL EXAMPLE**Fresh water at surface conditions thus has:  
    Hwater = 2.72 \* 10 ^ 26 \* (2 / 18) \* 62.4 = 18.83 \* 10 ^ 26 atoms/cuft  
For oil and water of the previous example:  
    Hwater = 2.72 \* 10 ^ 26 \* (2 / 18) \* 78.8 = 23.62 \* 10 ^ 26 atoms/cuft  
    Hoil = 2.72 \* 10 ^ 26 (2 / 16) \* 69.9 = 23.76 \* 10 ^ 26 atoms/cuft

**HYDROGEN CONTENT OF SOLIDS**The hydrogen content of solids, such as coal, can be computed from an equation similar to that for liquids.  
      1: Hcoal = 2.72 \* 10 ^ 26 \* (H / M) \* DENScoal

Hydrogen content is additive, as for densities, by summing the volume weighted hydrogen contents of each component. Thus:  
      2: Hmix = Sum (Hi \* Vi)

Where:  
  Hi = hydrogen content of a mixture  
  Hmix = hydrogen content of ith component  
  Vi = volume of ith component (fractional)

**NUMERICAL EXAMPLE**An average of a number of bituminous coals listed in a handbook shows the following analysis:  
      carbon     72.2% by weight  
      hydrogen   5.5% by weight  
      C2, N2 ,S    22.7% by weight  
  
The empirical chemical formula of this average coal is approximately C6H5.5O, and has a molecular weight of 96. The average density is 1.45 gm/cc.   
       Hcoal = 2.72 \* 10 ^ 26 \* (5.5 / 96) \* 1.45 \* 62.4 = 14.00 \* 10 ^ 26 atoms/cuft

Accurate data for the composition and density can be obtained from laboratory analysis and is preferable to average analysis such as this example.

Carnallite has the chemical formula KClMgCl2.6H20, with a molecular weight of 276, and a density of 1.57 gm/cc. Therefore:  
    Hcarn = 2.72 \* 10 ^ 26 \* (12 / 276) \* 1.57 \* 62.4 = 11.60 \* 10 ^ 26 atoms/cuft

**Apparent Neutron Log Porosity from Hydrogen Content**Since fresh water at surface conditions is defined as being 100% porosity and has a hydrogen concentration of 18.83 \* 10 ^ 26 atoms/cuft, the apparent porosity of other hydrogen-bearing substances can be calculated by direct proportion.  
      1: PHIN = Hmix / (18.83 \* 10 ^ 26)

Where:  
  Hmix = hydrogen concentration (atoms/cuft)  
  PHIN = apparent neutron log porosity (fractional)

**NUMERICAL EXAMPLE**The apparent neutron porosity of the gas, oil, water, coal, and carnallite from previous examples (at the temperature and pressure given earlier) are:  
      PHIN = Hmix / (18.83 \* 10 ^ 26)

|  |  |  |
| --- | --- | --- |
|  | Hmix | PHIN |
| Gas | 3.99\*10^26 | 0.21 |
| Water | 23.62\*10^26 | 1.25 |
| Oil | 23.76\*10^26 | 1.26 |
| Coal | 14.00\*10^26 | 0.74 |
| Carnallite | 11.60\*10^26 | 0.62 |

Neutron porosity values determined from hydrogen content are also additive.

This is a rather empirical approach to the problem, but it does give some idea of the amount of apparent porosity to expect, provided other neutron absorbers are absent or negligible.

**Neutron Response From Diffusion Length**  
Thermal neutron response in solids and fluids are related to a parameter called the diffusion length. The apparent limestone porosity of a rock versus diffusion length for thermal neutron logs of average intensity, is plotted below. It has been redrawn from "Radioactive Investigation of Oil and Gas Wells", translated from the Russian version by J. C. H. Muhlhaus, MacMillan Company, 1965.

  
*FIGURE 1: Neutron diffusion length versus porosity*

The diffusion length of a rock is calculated from:  
    1: Ld = 2.05 / (DENS \* (Sum (Qi / Mi) \* (Cci) \* Sum (Qi / Mi) (Csi)) ^ 1/2)

Where:  
  Cci = molecular capture cross section of the ith component (barns)  
  Csi = scattering cross section of the ith component (barns)  
  DENS = density (lb/cuft)  
  Ld = diffusion length (ft)  
  Mi = molecular weight of the ith component (unitless)  
  Qi = weight fraction of the ith component (unitless)

For chemical compounds, the formula reduces to:  
      2: Ld = 2.05 \* M / (DENS \* (Sum(Vi \* Cci) \* Sum(Vi \* Csi)) ^ 1/2)

Where:  
  M = molecular weight of the compound (unitless)  
  Vi = number of atoms of the ith element in the compound (unitless)

For elements the equation becomes:  
      3: Ld = 2.05 \* A / (DENS \* Cs)

Where:  
  A = the atomic number (unitless)  
  Cs = scattering cross section (barns)

Cross section values and summaries of data may be found in "Handbook of Physical Constants" edited by S.P. Clark, Geological Society of America, New York, 1966 and in the tables at the end of this Chapter.

**NUMERICAL EXAMPLE**

Calculate the diffusion length of the bituminous coal of the earlier example from its elemental values.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Vi | Cci | Csi | Vi\*Cci | Vi\*Csi |
| C | 6.0 | 0.0045 | 4.8 | 0.024 | 28.8 |
| H | 5.5 | 0.31 | 45.0 | 1.7 | 235.0 |
| O | 1.0 | 0.0016 | 4.1 | 0.0016 | 4.1 |
| TOTAL |  |  |  | 1.72 | 268 |

  DENS = 1.45 gm/cc \* 62.4 = 91 lb/cuft  
  M = 96  
  Ld = 2.05 \* 96 / (91 \* (1.72 \* 268) ^ 1/2) = 0.100 ft

This agrees closely with values given in tables and represents 65% apparent porosity on the chart in the graph shown in Figure 1. This compares with an apparent neutron log porosity of 74% found earlier for this coal by the hydrogen concentration method. Since the graph is rather flat in this region, the difference is not too hard to understand. As well, the chart only applies to one particular theoretical tool. The response curve for each real logging tool varies considerably from this ideal case. This approach is described in full in "Radioactive Investigations of Oil and Gas Wells", translated by J. O. H. Muhlhaus, MacMillan Company, 1965.

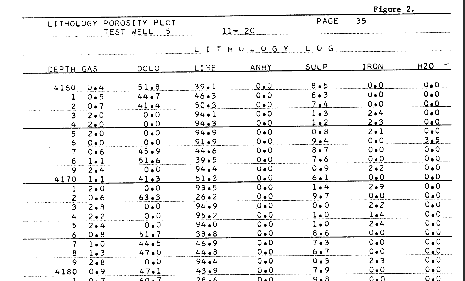
**A Comprehensive Example**

An example of simultaneous equation methods using data generated by these methods is shown in Figure 2. This is a solution of sonic, density and sidewall neutron information over a suspected gas zone in the Slave Point Formation in a well in northern Alberta.

The data used is presented in the following table:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| ROCKS | GAS | DCLO | LIME | ANHY | SULP | IRON | H20 |
| SYMBOLS | G | D | L | A | S | I | H |
| SONIC | 200.0 | 43.0 | 47.5 | 50.0 | 122.0 | 65.0 | 188.0 |
| DENSITY | 0.235 | 2.870 | 2.710 | 2.980 | 2.030 | 5.060 | 1.000 |
| PHIN\_LS | 0.500 | 0.020 | 0.000 | -0.005 | -0.030 | 0.000 | 1.000 |

While the example is somewhat simple, it does illustrate the fact that a log interpretation technique can be used in an unconventional manner. One should remember also that the logging devices utilized here are measuring properties of the flushed zone and any gas calculated represent unflushed gas.

  
*Figure 2: Sample printout from Sproule/Log, a program designed to use the basic physics concepts discussed in this paper. (CDC 3300. Fortran IV, 1969)*

**CONCLUSIONS**Methods are available for the prediction of sound velocity, density, and neutron log response in various environments which surround boreholes drilled for hydrocarbon or mineral exploration and production. The data necessary for these predictions are available from handbooks; however, considerable manipulation of the information is often necessary prior to use.

Calculations using these methods on familiar materials give answers in close agreement with expected results. It is therefore likely that use can be made of these methods in complex environments. It is to be hoped that this approach will stimulate further research and thought on the subject of the log response to its environment, and on the use of digital computers in the processing of log data.

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